MATH-9 TEST 4 (Unit 4 - Inverses, Logs & Exponentials)
SAMPLE

100 points NAME:____________

Be sure to clearly show your work on test. Unless specified, exact answers are required. Calculators are allowed, but no graphing calculators.

In problems 1-5, circle T for True, F for False (2 points each)

T F (1) The range of $\cos^{-1}(x)$ is $[-1,1]$. domain is $[-1,1]$ range is $[0,\pi]$

T F (2) $f(x) = |x|$ is a one-to one function.

T F (3) The domain of $f(x) = \log x$ is $[0,\infty)$. $(0,\infty)$

T F (4) $\ln\left(\frac{x\sqrt{y}}{z^5}\right)$ can be expanded as $\ln x + \frac{1}{2}\ln y - 5\ln z$.

T F (5) $\tan^{-1}(-\sqrt{3}) = -\pi/3$

In problems 6 - 10, fill in the blank with the most appropriate answer. (2 points each)

(6) $\log_3 27 + \log_7 1 + \log 100 =$ 5

$3 + 0 + 2$

(11) If $\log_b \frac{1}{16} = -2$ then b = 4

(7) $\sin^{-1}(1/2) =$ $\pi/6$

(12) $\log_2 56 - \log_2 7 =$ 3

$\log_2 8$

(8) $\ln 0 =$ undefined

(13) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ $5\pi/6$

(9) $3^{\log_3 12} =$ 12

(14) $\tan^{-1}(-1) =$ $-\pi/4$

(10) If $\log_8 x = 2/3$ then x = $8^{2/3} = 4$

(15) $\sin^{-1}(\sin(2\pi/3)) =$ $\pi/3$

(16) $\sin^{-1}(-1) =$ $-\pi/2$

(19) $\cos^{-1}(1) =$ 0

(17) $\log_{1/3} 1 =$ 0

(20) $3^{\log_3 x} =$ X

(18) Using your calculator, $\log_2 12 =$ $\frac{\log 12}{\log 2} \approx 3.585$ (to three decimal places).

(21) Find the domain of f(x) if $f(x) = \frac{1}{\log_3(x-2)}$, and express answer using interval notation. (4 pts)

$x - 2 > 0$ and denom $\neq 0$

$x > 2$

$\log_3(x-2) \neq 0$

$3^0 \neq x - 2$

$1 \neq x - 2$

$x \neq 3$

$(2,3) \cup (3,\infty)$

SOLVE THE FOLLOWING EXPONENTIAL AND LOGARITHMIC EQUATIONS EXACTLY, THEN GET AN APPROXIMATE ANSWER USING YOUR CALCULATOR. (7 points each)

(22) $\log x + \log(x+1) = \log 12$ must check

$\log(x(x+1)) = \log 12$

$x(x+1) = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = \cancel{-4}, 3$

$x = 3$

(23) $4^{2X-1} = 6.$

$\log 4^{2x-1} = \log 6$

$(2x-1)\log 4 = \log 6$

$2x - 1 = \frac{\log 6}{\log 4}$

$2x = \frac{\log 6}{\log 4} + 1$

$x = \frac{1}{2}\left(\frac{\log 6}{\log 4} + 1\right) \approx$ ________

(24) $\log_8(x+5) - \log_8(x-2) = 1$ must check

$\log_8 \frac{x+5}{x-2} = 1$

$\frac{x+5}{x-2} = 8^1$

$x+5 = 8(x-2)$

$x+5 = 8x - 16$

$21 = 7x$

$x = 3$ checks

(25) $e^{2x} - e^{x} - 2 = 0$

$(e^x - 2)(e^x + 1) = 0$

$e^x = 2 \qquad e^x = -1$

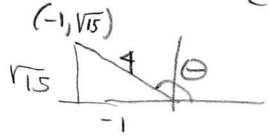
never

$\underline{x = \ln 2}$

(26) Find exactly (2 points each)

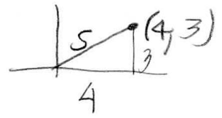
(a) $\sin(\cos^{-1}(-1/4)) = \sin\theta = \frac{\sqrt{15}}{4}$

$\theta = \cos^{-1}\left(-\frac{1}{4}\right)$



(b) $\cos(2\underbrace{\tan^{-1}(3/4)}_{\theta}). = \cos 2\theta = 2\cos^2\theta - 1$

$\theta = \tan^{-1}\frac{3}{4}$



$= 2\left(\frac{4}{5}\right)^2 - 1$

$= 2 \cdot \frac{16}{25} - 1$

$= \frac{7}{25}$

(27) (a) Find all (exact) solutions in [0, 2π) : cos ß = 2/3 (4 points)

$\cos^{-1}(2/3)$

$2\pi - \cos^{-1}(2/3)$

(b) Find all (exact) solutions in [0, 2π] : sin ß = -1/4 (4 points)

$\pi + \sin^{-1}(1/4)$ $\quad 2\pi - \sin^{-1}(1/4)$

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(28) Given f(x) = $\log_2(x-2)-1$, find the following. Pay close attention to details, show all steps precisely.

(a) $f^{-1}(x)$ (6 points)

(b) domain and range of f(x) and $f^{-1}(x)$. (4 points)

(c) graph f(x) and $f^{-1}(x)$ (6 points)

$f(x) = \log_2(x-2) - 1$

$y = \log_2(x-2) - 1$

switch

$x = \log_2(y-2) - 1$

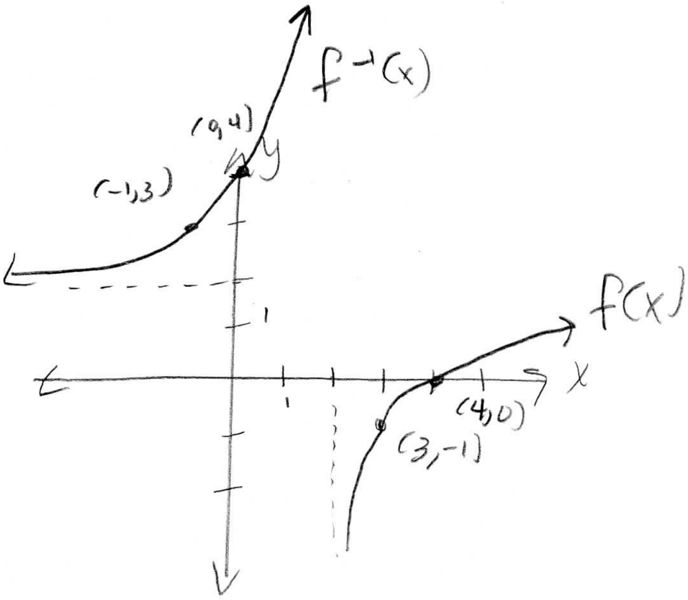
$x+1 = \log_2(y-2)$

$2^{x+1} = y - 2$

$y = 2^{x+1} + 2$

So

$f^{-1}(x) = 2^{x+1} + 2$



	f	f^{-1}
domain	$(2,\infty)$	$(-\infty,\infty)$
range	$(-\infty,\infty)$	$(2,\infty)$