on graphs
Show scale and
Label points if asked

MATH-9 TEST 3 (Unit 3 - Functions and Graphs)

SAMPLE

100 points NAME:______________

Show all work on the test. On graphs, you are expected to use your knowledge of shifting etc. as opposed to simply plotting points. Be sure to clearly show scale. No graphing calculators.

Fill in the blanks. (2 points each)

(1) To obtain the graph of f(x) = $\sqrt{x-2}+3$ we can shift the graph of g(x) = $\sqrt{x}$ (how many units, which way?) 2 right, 3 up

(2) The domain of f(x) = cotx is $x \neq \pi k$

(3) Is $x^2 + y^2 = 4$ an example of a function? NO

(4) is $f(x) = x^3\sin x$ even, odd, or neither? even.

(5) The range of f(x) = 3sin(x) is $[-3, 3]$

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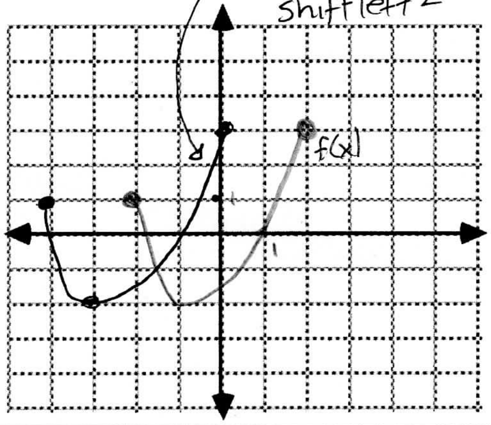
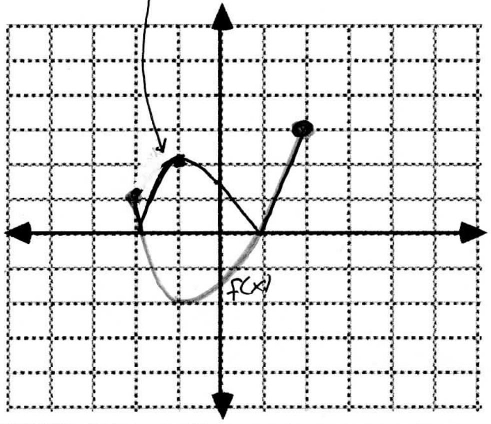
(6) Given the graph of y = f(x) as shown on both graphs below, (10 points)

(a) Find: Domain $[-2,2]$ Range $[-2,3]$.

(b) Graph (a) y = f(x+2) Shift left 2 (b) y = | f(x)| . "Bottom flips up"

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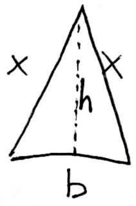
(7) An isosceles triangle has a perimeter of 8 cm. Express the area of the triangle as a function of the length b of the base of the triangle. Simplify (6 points)




Area= $\frac{1}{2}bh$

Need h in terms of b.

Know $(\frac{1}{2}b)^2+h^2=x^2$ so $h^2 = x^2 - \frac{1}{4}b^2$

what is x? Know $2x+b=8$ (perimeter) so

$x = \frac{8-b}{2}$ so $h = \sqrt{\left(\frac{8-b}{2}\right)^2 - \frac{1}{4}b^2} = \sqrt{32-8b}$

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(8) Given $f(x) = 3x^2 - 2$ find and simplify: (7 points)

$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2-2-(3x^2-2)}{h}$

$= \frac{3x^2+6xh+3h^2-2-3x^2+2}{h} = \frac{6xh+3h^2}{h} = 6x+3h$

(9) Find the domain for each of the following functions (4 points each)

(a) $f(x) = \dfrac{\sqrt{2-x}}{x^2 - x - 12}$

denom. $\neq 0$ $x^2 - x - 12 \neq 0$

$(x-4)(x+3) \neq 0$

Also $x \neq 4, -3$

radicand $\geq 0$ $2 - x \geq 0$

$-3$ $x \leq 2$

Put together

$2$ $(-\infty, -3) \cup (-3, 2]$

(b) $g(x) = \dfrac{x}{\cos 4x - 1}$

denom. $\neq 0 \Rightarrow$

$\cos 4x - 1 \neq 0$

$\cos 4x \neq 1$

$4x \neq 2\pi k$

$x \neq \frac{\pi}{2} k$

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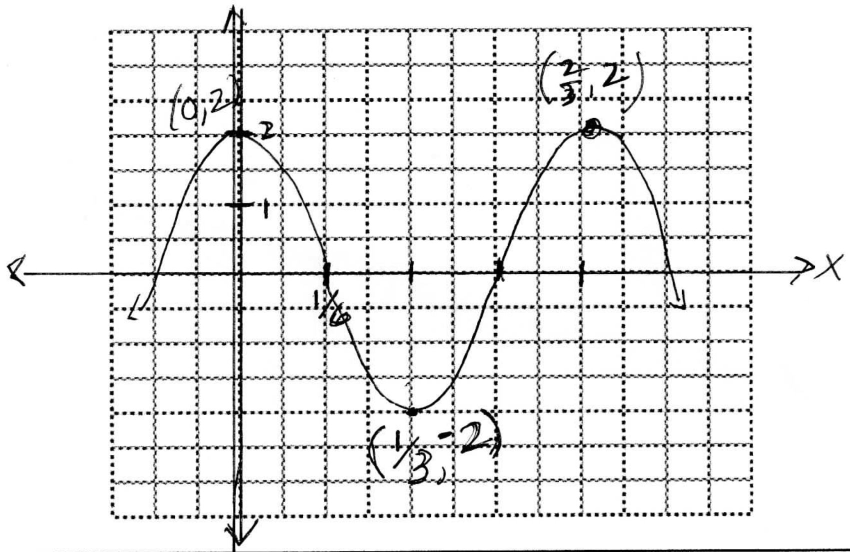
(10) Graph $f(x) = 2\cos(3\pi x)$. Label coordinates of highs and lows. (8 points)




period $= \frac{2\pi}{3\pi} = \frac{2}{3}$

quarter period $\frac{1}{4} \cdot \frac{2}{3} = \frac{1}{6}$

Good idea

Check a point $(\frac{1}{3}, -2)$

$f(\frac{1}{3}) = 2\cos(3\pi \cdot \frac{1}{3}) = 2\cos\pi = -2$ ✓

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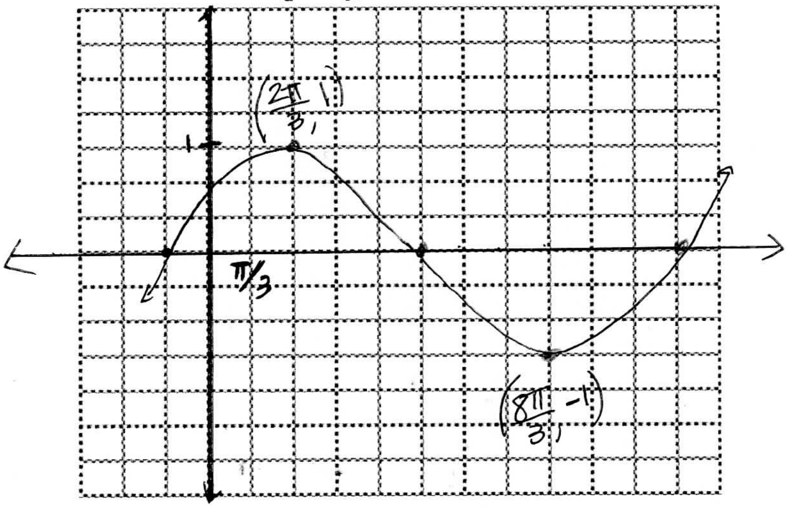
(11) Graph $f(x) = \sin\left(\frac{1}{2}x + \frac{\pi}{6}\right)$ Label coordinates of highs and lows. (10 points)




$\sin\left[\frac{1}{2}(x + \frac{\pi}{3})\right]$

period $= \frac{2\pi}{1/2} = 4\pi$

Quarter period $= \pi$

shift $= \frac{\pi}{3}$ left

scale: 1 square $= \frac{\pi}{3}$ so

quarter period is 3 squares

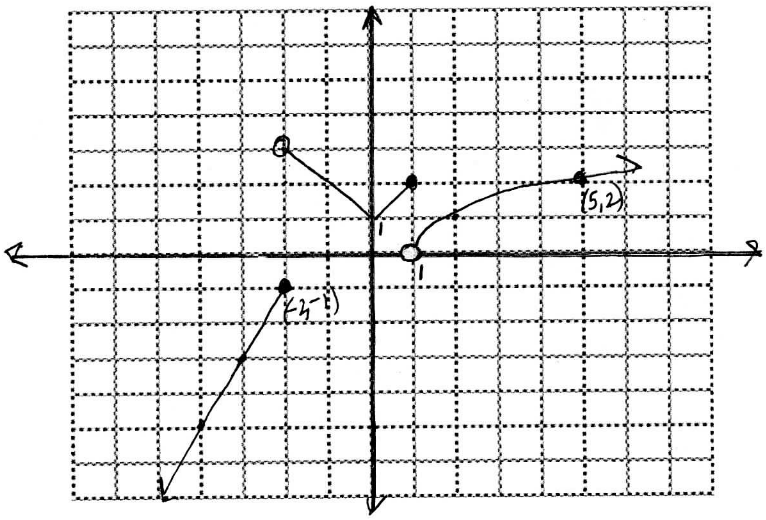
Check a point $(\frac{8\pi}{3}, -1)$

$f(\frac{8\pi}{3}) = \sin(\frac{1}{2} \cdot \frac{8\pi}{3} + \frac{\pi}{6})$

$= \sin(\frac{8\pi}{6} + \frac{\pi}{6}) = \sin(\frac{9\pi}{6})$

$= \sin(\frac{3\pi}{2}) = -1$ ✓

(12) Graph $f(x) = \begin{cases} 2x+3 & \text{if } x \le -2 \\ |x|+1 & \text{if } -2 < x \le 1 \\ \sqrt{x-1} & \text{if } x > 1 \end{cases}$ (10 points)

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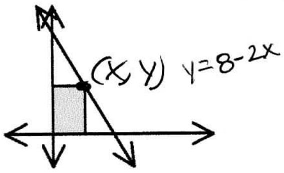
(13) The point P lies in the first quadrant on the graph of the line y = 8 - 2x. From the point P, perpendiculars are drawn to both the x-axis and the y-axis. What is the largest possible area for the rectangle thus formed? (10 points)




Maximize Area $= xy$

$A = x(8-2x)$

$A = 8x - 2x^2$

Maximum at vertex.

$x = \frac{-b}{2a} = \frac{-8}{2(-2)} = 2$

Max Area $= A(2) = 2(8-2(2)) = 8$ sq units

give units

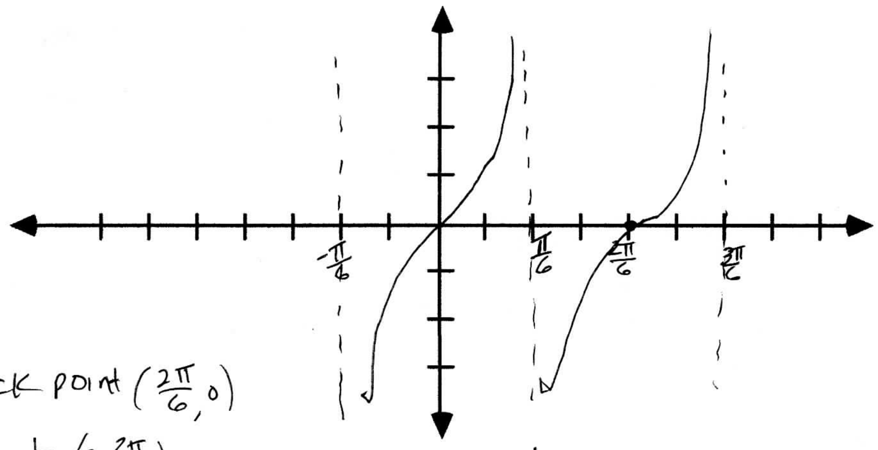
Make sure to answer question asked.... Sometimes asked dimensions, not area

dimensions would be

$x = 2$ units

$y = 8-2x = 8-2(2) = 4$ units

(14) Graph f(x) =tan(3x). Two periods. Show asymptotes. (8 points)




double check period $= \frac{\pi}{3}$

check point $\left(\frac{2\pi}{6}, 0\right)$

$f\left(\frac{2\pi}{6}\right) = \tan\left(3 \cdot \frac{2\pi}{6}\right)$

$= \tan \pi = 0$ ✓

not $2\pi$

Period $= \frac{\pi}{3}$

Asymptotes where $\tan(3x)$ undefined so when

$\cos(3x) = 0$

$3x = \frac{\pi}{2} + \pi k$

$x = \frac{\pi}{6} + \frac{\pi}{3}k$

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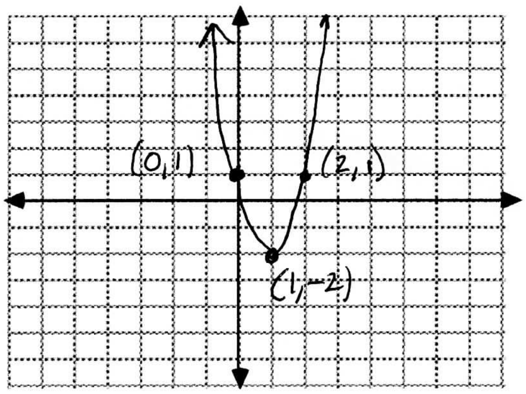
(15) Given the function $f(x) = 3x^2 - 6x + 1$
put f(x) in the form $f(x) = a(x-h)^2 + k$ and sketch the graph. On the graph label the vertex plus one other point.

( 8 points)




$f(x) = 3(x^2 - 2x \quad ) + 1$

$= 3(x^2 - 2x + 1) + 1 - 3$

$f(x) = 3(x-1)^2 - 2$

vertex $(1, -2)$

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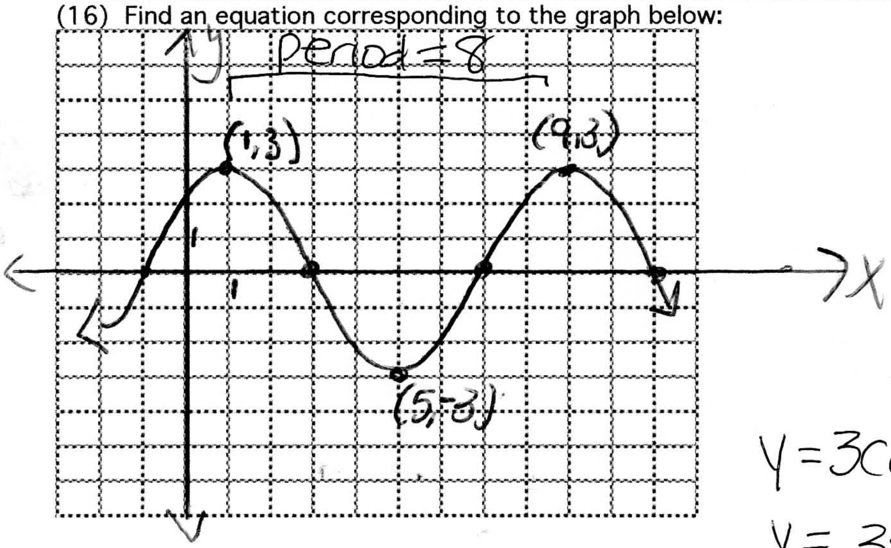
(16) Find an equation corresponding to the graph below: (5 points)




Amplitude=3

period=8 $\Rightarrow \frac{2\pi}{b} = 8$

$\Rightarrow b = \frac{\pi}{4}$

$y = \pm 3 \begin{matrix}\cos \\ \sin\end{matrix} \left[\frac{\pi}{4}(x \pm c)\right]$

$y = 3\cos\left[\frac{\pi}{4}(x-1)\right] = 3\cos\left(\frac{\pi}{4}x - \frac{\pi}{4}\right)$

$y = 3\sin\left[\frac{\pi}{4}(x+1)\right] = 3\sin\left(\frac{\pi}{4}x + \frac{\pi}{4}\right)$

other possibilities

Check a point.