Solving Exponential Equations

We have discussed three methods to use when solving exponential equations.

1) IF both sides of the equation can be written having the same base, we can use the one-to-one property of the exponential function and equate the exponents. (If $a^x = a^y$ then $x=y$)

Ex. $3^{2x-1} = 27$
=> $3^{2x-1} = 3^3$
=> $2x-1=3$
=> $x=2$

2) We can use the definition of logarithmic notation($y=\log_a x$ means $a^y=x$) to rewrite the log in exponential form, thus isolating the variable.

Ex. $5^x = 12$
=> $x=\log_5 12$

3) We can use the properties of logarithms to bring the exponent down.

Ex. $5^x = 12$ Take the log or ln of both sides.
=> $\log 5^x = \log 12$
=> $x\log 5 = \log 12$
=> $x= \frac{\log 12}{\log 5} (exact) \approx 1.54$

Ex. $7^{x+3} = 8$ Take the log or ln of both sides.
=> $\ln 7^{x+3} = \ln 8$
=> $(x+3) \ln 7 = \ln 8$
=> $x\ln 7 + 3\ln 7 = \ln 8$
=> $x\ln 7 = \ln 8 - 3\ln 7$
=> $x = \frac{\ln 8 - 3\ln 7}{\ln 7} (exact) \approx -1.93$

Ex. $4^{2-5x} = 6^x$ Take the log or ln of both sides.
=> $\ln 4^{2-5x} = \ln 6^x$
=> $(2-5x) \ln 4 = x \ln 6$
=> $2\ln 4 – 5x\ln 4 = x\ln 6$ Gather terms with x on one side.
=> $2\ln 4 = 5x \ln 4 + x \ln 6$ Factor out the x.
=> $2 \ln 4 = x(5\ln 4 + \ln 6)$
=> $x = \frac{2\ln 4}{5\ln 4 + \ln 6} (exact) \approx 0.32$

Solving Logarithmic Equations

We have discussed two methods to use when solving logarithmic equations.

1) If the equation contains <u>only</u> logarithmic terms, each having the same base, we use the one-to-one property of logarithms ($\log_a x = \log_a y \Rightarrow x = y$) to equate the arguments.

Ex. $\log_3(x-1) = \log_3 12$

$\Rightarrow$ $x-1=12$

$\Rightarrow$ $x=13$

We may have to first use the properties of logarithms to first obtain a <u>single</u> logarithmic term on each side. (Cannot simply "cancel logs')

Ex. $\log_5(2x) - \log_5(7) = \log_5(x+1)$

$\Rightarrow$ $\log_5\left(\frac{2x}{7}\right) = \log_5(x+1)$

$\Rightarrow$ $\frac{2x}{7} = x+1$

$\Rightarrow$ $2x=7(x+1)$

$\Rightarrow$ $-5x=7$

$\Rightarrow$ $x= -7/5$

****REMEMBER, we <u>must check</u> the answer to logarithmic equations. In this case, $x= -7/5$ makes the argument of the logarithm negative so it does not work. The answer here is NO SOLUTION.

2) If the equation contains logarithmic terms, each having the same base, AND terms without a log we use the definition of logarithms to rewrite the log as an exponent($y=\log_a x$ means $a^y=x$).

Ex. $\log(x+5)= 3$

$\Rightarrow$ $10^3 = x+5$

$\Rightarrow$ $1000=x+5$

$\Rightarrow$ $x=995$

We may have to first use the properties of logarithms to first obtain a <u>single</u> logarithmic term on one side and a single number on the other.

Ex. $\log_6(3+x) + \log_6(x+4) =1$

$\Rightarrow$ $\log_6[(3+x)(x+4)]=1$

$\Rightarrow$ $6^1=(3+x)(x+4)$

$\Rightarrow$ $6=x^2+7x+12$

$\Rightarrow$ $x^2+7x+6=0$

$\Rightarrow$ $(x+6)(x+1) = 0$

$\Rightarrow$ $x= -6$, $x= -1$…. but $x= -6$ doesn't check.

$\Rightarrow$ $x= -1$