Practice Integration Problems
MATH 182: Fall 2006

The integrals practice problems on the following pages can all be evaluated using combinations of

1) The Method of Substitution

2) Integration by Parts

3) Trigonometric identities

4) Inverse Trigonometric Substitutions

5) Partial fraction expansions

Some commonly used trigonometric identities are:

$$
\begin{aligned}
sin^2(x) + cos^2(x) &= 1 \\
tan^2(x) + 1 &= sec^2(x) \\
cos^2(x) &= \frac{1}{2}\left(1 + cos(2x)\right) \\
sin^2(x) &= \frac{1}{2}\left(1 - cos(2x)\right) \\
sin(2x) &= 2\ sin(x)\ cos(x) \\
sin(x)\ cos(y) &= \frac{1}{2}\left(sin(x+y) + sin(x-y)\right) \\
cos(x)\ cos(y) &= \frac{1}{2}\left(cos(x+y) + cos(x-y)\right) \\
sin(x)\ sin(y) &= \frac{1}{2}\left(cos(x-y) - cos(x+y)\right)
\end{aligned}
$$

Some commonly integrals worth noting include:

$$
\begin{aligned}
\int \frac{1}{u^2 + 1}\ du &= arctan(u) + c \\
\int \frac{1}{\sqrt{1 - u^2}}\ du &= arcsin(u) + c \\
\int tan(u)\ du &= ln|sec(u)| + c \\
\int cot(u)\ du &= -ln|csc(u)| + c \\
\int sec(u)\ du &= ln\,|sec(u) + tan(u)| + c \\
\int csc(u)\ du &= ln\,|csc(u) - cot(u)| + c \\
\int sec^3(u)\ du &= \frac{1}{2}\left(sec(x)\ tan(x) + ln\,(sec(x) + tan(x))\right) + c
\end{aligned}
$$

(I) Quickies:

a) $\int 5e^{3x} \ dx = \frac{5}{3}e^{3x} + c$

b) $\int 2\cos(\pi x) \ dx = \frac{2}{\pi}\sin(\pi x) + c$

c) $\int \sec(2x) \ \tan(2x) \ dx = \frac{1}{2}\sec(2x) + c$

d) $\int 7\sec^2(5x) \ dx = \frac{7}{5} \ \tan(5x) + c$

e) $\int \frac{dx}{x^2+4} = \frac{1}{2}arctan\left(\frac{x}{2}\right) + c$

f) $\int \frac{x}{x^2+1} \ dx = \frac{1}{2}ln\left|x^2 + 1\right| + c$

g) $\int \frac{2}{3x+1} \ dx = \frac{2}{3} \ ln|3x + 1| + c$

(II) Intermediate Difficulty Problems:

1) $\int_1^2 \frac{\ln(x)}{x} \ dx$

2) $\int_1^2 \frac{\ln(x)}{x^2} \ dx$

3) $\int \frac{2x+1}{x(1-x)} \ dx$

4) $\int xe^{x/2} \ dx$

5) $\int \frac{e^{\sqrt{z}}}{\sqrt{z}} \ dz$

6) $\int \tan^3(x) \ \sec^2(x) \ dx$

7) $\int \frac{dx}{\sqrt{9-x^2}}$

8) $\int \frac{dx}{\sqrt{x^2-9}}$

9) $\int \frac{3x+2}{x^2(x+2)} \ dx$

10) $\int \frac{x^3}{1+x^4} \ dx$

11) $\int \frac{3x^3+x^2+4}{3x+1} \ dx$

12) $\int_0^{\pi/2} \sin^2(x) \ dx$

13) $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)} \ dx$

14) $\int (2x + 1)\cos(x) \ dx$

15) $\int e^{3x} \ \cos(4x) \ dx$

16) $\int \frac{1}{x\sqrt{1+x^2}} \ dx$

17) $\int_0^1 arcsin(x) \ dx$

18) $\int_0^{\pi/6} \frac{\cos(x)}{1+\sin(x)} \ dx$

19) $\int \sqrt{x^2 - 4} \ dx$ *(need table)*

20) $\int \sqrt{4 - x^2} \ dx$

21) $\int \frac{4x+7}{(x+1)(2x+3)} \, dx$

22) $\int \frac{x}{\sqrt{1+x^2}} \, dx$

23) $\int \frac{sin(ln(x))}{x} \, dx$

24) $\int x^2 e^x \, dx$

25) $\int sec^4(x) \, dx$

26) $\int \frac{e^x}{e^{2x}+1} \, dx$

27) $\int cos^2(4x) \, dx$

28) $\int cos^2(x) \, sin^3(x) \, dx$

29) $\int_{\pi/4}^{\pi/3} \frac{sec^2(x)}{tan(x)} \, dx$

30) $\int arctan(2x) \, dx$

31) $\int \frac{1}{x^2+4x+5} \, dx$

32) $\int sin(2x) \, cos(4x) \, dx$

33) $\int \frac{4x^2-2x}{(x-1)(x^2+1)} \, dx$

34) $\int \frac{1}{(x^2+4)^{3/2}} \, dx$

35) $\int x^2 \, ln(x) \, dx$

36) $\int x^2 e^{x^3} \, dx$

37) $\int tan(x) \, sec^3(x) \, dx$

38) $\int \frac{x}{\sqrt{1+x^2}} \, dx$

39) $\int \frac{2x+1}{x^2-1} \, dx$

40) $\int sin(x) \, cos^3(x) \, dx$

41) $\int \frac{1}{\sqrt{x^2+2x+2}} \, dx$

42) $\int \frac{3 \, cos(x)}{\sqrt{1+3sin(x)}} \, dx$

Answers:

1) $u = ln(x)$; $\frac{1}{2}(ln(2))^2$

2) $u = ln(x), v = -x^{-1}$; $1/2(1 - ln(2))$

3) $partial\ fraction$; $ln|x| - 3ln|x - 1| + c$

4) $u = x, v = 2e^{x/2}$; $2(x - 2)e^{x/2} + c$

5) $u = \sqrt{x}$; $2e^{\sqrt{x}} + c$

6) $u = tan(x)$; $1/4\ tan^4(x) + c$

7) $u = x/3; arcsin(x/3) + c$

8) $x = 3\ sec(\theta); ln|\frac{1}{3}x + \frac{1}{3}\sqrt{x^2 - 9}| + c$

9) $partial\ fractions; ln|x| - ln|x + 2| - x^{-1} + c$

10) $u = 1 + x^4; 1/4\ ln(x^4 + 1) + c$

11) $Long\ division; 1/3x^3 + 4/3\ ln|3x + 1| + c$

12) $trig\ ident.; \pi/4$

13) $u = \sqrt{x} + 1; 2ln(1 + \sqrt{x}) + c$

14) $u = 2x + 1, v = sin(x); 2cos(x) + (2x + 1)\ sin(x) + c$

15) $IBParts\ twice; 3/25e^{3x}cos(4x) + 4/25e^{3x}sin(4x) + c$

16) $x = tan(\theta); -ln|(1 + \sqrt{1 + x^2})/x| + c$

17) $u = arcsin(x), v = x; \pi/2 - 1$

18) $u = 1 + sin(x); ln(3) - ln(2)$

19) $x = 2sec(\theta); \frac{1}{2}\ x\sqrt{x^2 - 4} - 2ln|x + \sqrt{x^2 - 4}| + c$

20) $x = 2sin(\theta); 1/2x\sqrt{4 - x^2} + 2arcsin(x/2) + c$

21) $partial\ fraction; 3ln|x + 1| - ln|2x + 3| + c$

22) $u = 1 + x^2; \sqrt{1 + x^2}$

23) $u = ln(x); -cos(ln|x|) + c$

24) $IBPtwice, u = x^2, v = e^x; (2 - 2x + x^2)e^x$

25) $trig.ident.\ then u = tan(x); tan(x) + \frac{1}{3}tan^3(x) + c$

26) $u = e^x; arctan(e^x) + c$

27) $trig.\ ident.; 1/2x + 1/6 * sin(8x) + c$

28) $trig.ident,\ u = cos(x); 1/5cos^5(x) - 1/3cos^3(x) + c$

29) $u = tan(x); 1/2ln(3) = ln(\sqrt{3})$

30) $IBP\ u = arctan(2x), v = x; x\arctan(2x) - 1/4ln(1 + 4$

31) $u = x + 2, complete\ square; arctan(x + 2) + c$

32) $trig.\ ident.; 1/4cos(2x) - 1/12cos(6x) + c$

33) $Partial.\ Frac.; ln|x - 1| + arctan(x) + 3/2ln(x^2 + 1) + c$

34) $x = 2tan(\theta); x/(4\sqrt{x^2 + 4}) + c$

35) $IBP\ u = ln(x), v = 1/3x^3; 1/3x^3ln|x| - 1/9x^3 + c$

36) $u = x^3; 1/3e^{x^3} + c$

37) $trig.\ u = sec(x); 1/3\ sec^3(x) + c$

38) $u = 1 + x^2; \sqrt{1 + x^2} + c$

39) $partial\ frac.; 3/2ln|x - 1| + 1/2ln|x + 1| + c$

40) $u = cos(x); -1/4cos^4(x) + c$

41) $x + 1 = tan(\theta); ln|x + 1 + \sqrt{x^2 + 2x + 2}| + c$

42) $u = 1 + 3sin(x); 2\sqrt{1 + 3sin(x)} + c$