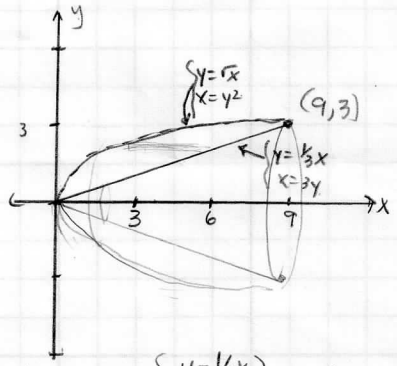
5A Volume Worksheet - Solutions

(2)



Graphs intersection $\begin{cases} y=\frac{1}{3}x \\ y=\sqrt{x} \end{cases} \Rightarrow \frac{1}{3}x=\sqrt{x}$

$x^2=9x$

$x^2-9x=0$

$x=0,9$

a) SHELLS

draw rectangle parallel to x-axis $\Rightarrow$ dy $2\pi r h$ ($r=y$, $h=3y-y^2$)

$$V=2\pi\int_0^3 y(3y-y^2)\,dy$$

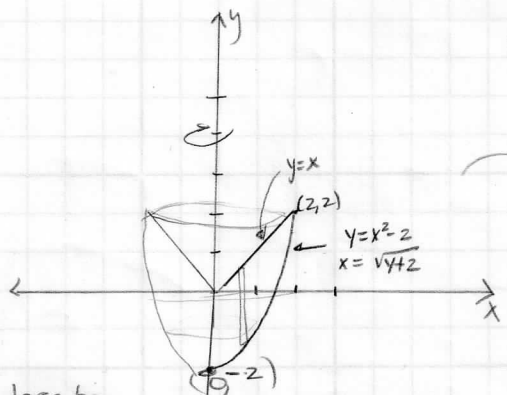
b) Disks/Washers

typical slice is washer so $\pi(R_o^2-R_i^2)$ $\quad R_o=\sqrt{x}$, $R_i=\frac{1}{3}x$

$$V=\pi\int_0^9\left[(\sqrt{x})^2-(\tfrac{1}{3}x)^2\right]dx$$

ANS Volume $=\frac{27\pi}{2}$ cu. units

(3)



Intersection $\begin{cases} y=x \\ y=x^2-2 \end{cases} \rightarrow x=x^2-2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,-1$

(a) wrt x ... dx ... implies shell method

draw rectangle parallel to y-axis $2\pi r h$ ($r=x$, $h=x-(x^2-2)$)

$$V=2\pi\int_0^2 x\left[x-(x^2-2)\right]dx$$

(b) wrt y ... dy $\Rightarrow$ disks/washers

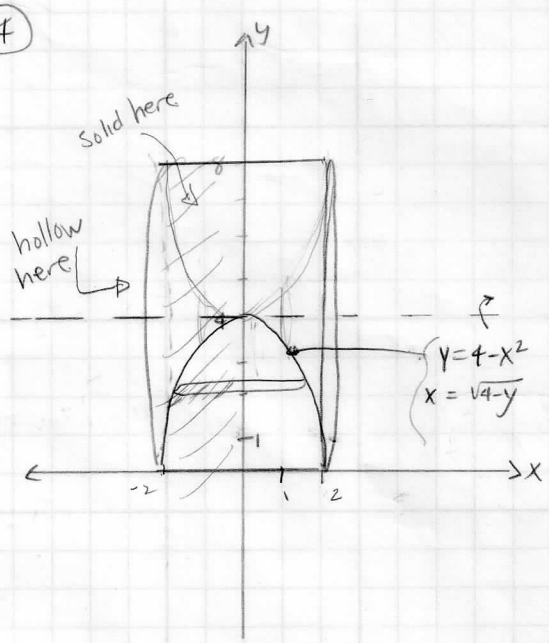
Must break into 2 pieces ... bottom is solid, top is hollow

$$\pi\int_{-2}^{0}(\sqrt{y+2})^2\,dy+\pi\int_0^2\left((\sqrt{y+2})^2-y^2\right)dy$$

("R^2") ("$R_o^2-R_i^2$")

Ans. $V=\frac{16\pi}{3}$ cu. units

(4)



a) Shells Use symmetry - consider right half & double it.

draw rectangle parallel to $y=4$

$2\pi r h$ ($r=4-y$, $h=\sqrt{4-y}$)

$$V=2\cdot 2\pi\int_0^4(4-y)\sqrt{4-y}\,dy=2\cdot 2\pi\int_0^4(4-y)^{3/2}\,dy$$

b) Disks/Washers

Typical slice is washer so $\pi(R_o^2-R_i^2)$ ($R_o=4$, $R_i^2=(4-(4-x^2))^2$)

$$V=2\pi\int_0^2\left[4^2-(4-(4-x^2))^2\right]dx$$

Ans. $\frac{256\pi}{5}$