MATH-131 SAMPLE TEST 5 (6.8-8.2)

Summer 2014

100 points NAME: Solns

Show all work on the test. On this exam, allow for complex numbers.

Fill in the blanks.

(1) $(5+2i)(4-i)$ $22+3i$ $\quad 20+3i-2i^2 = 20+3i+2$

(2) $\log_7(1)$ 0

(3) $\log_3(81) =$ 4

(4) $i^{18} =$ -1 $\quad (i^{16})(i^2) = (i^4)^4 i^2 = (1)^4(-1)$

(5) $\log_{\frac{1}{2}}(16) =$ -4

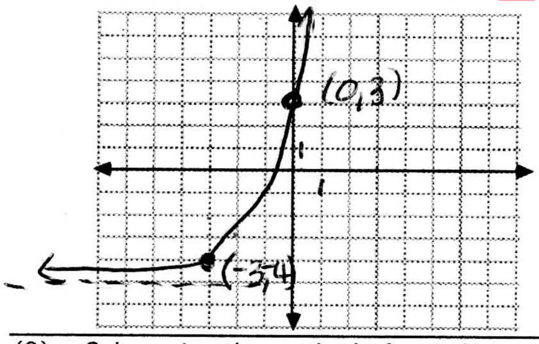
(6) The domain of $f(x)=e^x$ is $(-\infty, \infty)$

(7) Simplify $\frac{3+\sqrt{-36}}{6} =$ $\frac{1+2i}{2}$ $\quad \frac{3+i\sqrt{36}}{6} = \frac{3+i6}{6} = \frac{1+2i}{2}$

(8) Graph $f(x) = 2^{x+3} - 5$

a) What is the domain of f? $(-\infty, \infty)$

b) What is the range of f? $(-5, \infty)$



(9) Solve using the method of completing the square. (No credit given for using another method)

$-3x^2-2x+4=0$

or divide by -3 first

$$-3\left(x^2+\tfrac{2}{3}x \quad\right) = -4$$

$$-3\left(x^2+\tfrac{2}{3}x+\left(\tfrac{1}{3}\right)^2\right) = -4 + -\tfrac{1}{3}$$

$$-3\left(x+\tfrac{1}{3}\right)^2 = -\tfrac{13}{3}$$

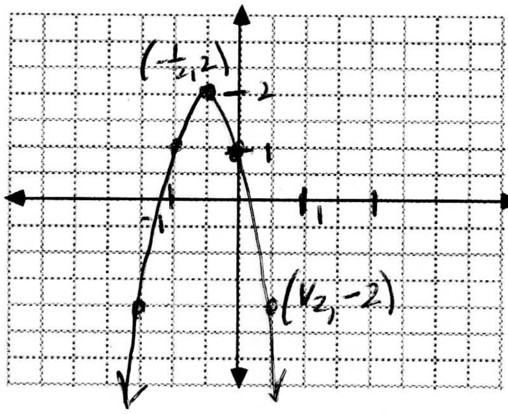
$$\left(x+\tfrac{1}{3}\right)^2 = +\tfrac{13}{9}$$

$$x+\tfrac{1}{3} = \pm\tfrac{\sqrt{13}}{3}$$

$$x = -\tfrac{1}{3} \pm \tfrac{\sqrt{13}}{3}$$

(10) Given the function $f(x) = -4x^2 - 4x + 1$

put $f(x)$ in the form $f(x) = a(x-h)^2 + k$ and sketch the graph. On the graph label the vertex plus one other point.



$f(x) = -4(x^2 + x \quad) + 1$

$= -4(\underbrace{x^2 + x + \tfrac{1}{4}}) + 1 + 1$
-1

$= -4(x + \tfrac{1}{2})^2 + 2$

Vertex: $(-1/2, 2)$

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(11) Given $f(x) = 3x^2 + 4x + 1$

a) Does this function have a maximum or a minimum value? (which) minimum

b) What is that value? $-1/3$

Vertex: $X = \frac{-b}{2a} = \frac{-4}{2(3)} = \frac{-4}{6} = \frac{-2}{3}$ $f(-2/3) = 3\left(\frac{4}{9}\right) + 4\left(\frac{-2}{3}\right) + 1 = \frac{4}{3} - \frac{8}{3} + 1 = \frac{-1}{3}$

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(13) For the following one-to-one functions, find $f^{-1}(x)$

(a) $f: \{(2,1),(3,5),(-1,7)\}$

f^{-1} $\{(1,2)(5,3)(7,-1)\}$

(b) $f(x) = 5 - 7x$

$y = 5 - 7x$ switch
$x = 5 - 7y$
$7y = 5 - x$
$y = \frac{5-x}{7}$ $f^{-1}(x) = \frac{5-x}{7}$

(b) $f(x) = \frac{3x}{x-1}$

$y = \frac{3x}{x-1}$ switch
$x = \frac{3y}{y-1}$
$x(y-1) = 3y$
$xy - x = 3y$
$xy - 3y = x$
$y(x-3) = x$
$y = \frac{x}{x-3}$
$f^{-1}(x) = \frac{x}{x-3}$

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(14) Given $f(x) = \sqrt{x+4}$; $g(x) = 3x^2 + 2$, find

(a) $(f \circ g)(x)$

$= f(g(x)) = f(3x^2+2)$
$= \sqrt{3x^2+2+4}$
$= \sqrt{3x^2+6}$

(b) $(g \circ f)(x)$

$= g(f(x)) = g(\sqrt{x+4})$
$= 3(\sqrt{x+4})^2 + 2$
$= 3(x+4) + 2$
$= 3x + 14$

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(15) Solve each of the following equations :

(a) $\log_b(81) = 4$

$b^4 = 81$
$b = 3$

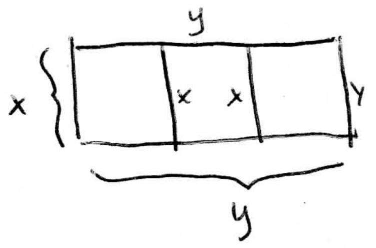
(b) $\log_5\left(\frac{1}{125}\right) = x$

$5^x = \frac{1}{125}$
$x = -3$

(c) $\log_9(27) = z$

$9^z = 27$
$(3^2)^z = 3^3$
$3^{2z} = 3^3$
$2z = 3$
$z = 3/2$

(16) A man wishes to put a fence around a rectangular field and then subdivide the field into three smaller rectangular plots by placing two fences parallel to one of the sides. If he can only afford 40 yards of fencing, what is the maximum area he can enclose?



Maximize Area = XY

Need a relationship between X and Y so we can get area as function of X only.

$4x + 2y = 40$

$2y = 40 - 4x$

$Y = 20 - 2X$

So Area $= x(20 - 2x)$

$A = 20x - 2x^2$ Max at vertex $X = \frac{-b}{2a} = \frac{-20}{2(-2)}$

$= 5$

So max Area is

$A = 20X - 2x^2$

5

$A = 20(5) - 2(5)^2$

$= 100 - 50$

$= 50$ sq yds

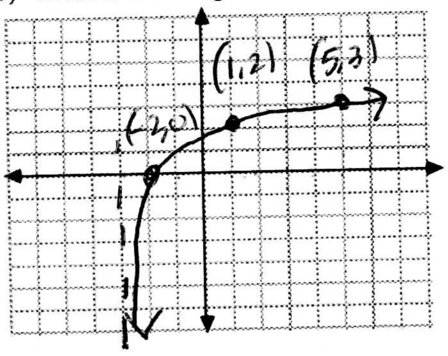
(17) Graph $f(x) = \log_2(x+3)$

a) What is the domain of f? $(-3, \infty)$

$X + 3 > 0$

$X > -3$

b) What is the range of f? $(-\infty, \infty)$



(18) Solve each of the following and simplify your answer:

(a) $5x^2=1-3x$

$5x^2+3x-1=0$

$x=\frac{-3\pm\sqrt{9+20}}{10}=\frac{-3\pm\sqrt{29}}{10}$

(b) $x^2+4x+9=0$

$x=\frac{-4\pm\sqrt{16-36}}{2}=\frac{-4\pm\sqrt{-20}}{2}$

$=\frac{-4\pm i\sqrt{20}}{2}=\frac{-4\pm 2i\sqrt{5}}{2}$

$=-2\pm i\sqrt{5}$

(c) $x^2\left(\frac{2}{x^2}-\frac{14}{x}+24\right)=(0)x^2$

$2-14x+24x^2=0$

$2(12x^2-7x+1)=0$

$2(3x-1)(4x-1)=0$

$x=1/3,\ 1/4$

(d) $(x-3)(x+1)=2$

$x^2-2x-3=2$

$x^2-2x-5=0$

$x=\frac{2\pm\sqrt{4+20}}{2}=\frac{2\pm\sqrt{24}}{2}$

$=\frac{2\pm 2\sqrt{6}}{2}=1\pm\sqrt{6}$

(e) $2x^{-2}+5x^{-1}-3=0$

let $x^{-1}=u$

$2u^2+5u-3=0$

$(2u-1)(u+3)=0$

$u=1/2,\ -3$

$\frac{1}{x}=\frac{1}{2}$ $\quad$ $\frac{1}{x}=-3$

$x=2$ $\quad$ $x=-1/3$

(f) $(1-3x)^2=-4$

$1-3x=\pm\sqrt{-4}=\pm 2i$

$-3x=-1\pm 2i$

$x=\frac{-1\pm 2i}{-3}$