Radicals -(Chapter 6)

SQUARE ROOT REVIEW:

Getting Ready for Radicals (pg 473+)

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ……..

Square Root: *b is a square root of a if* $b^2=a$

Examples:

Square root(s) of 25: _________ Note: every positive number has ___ square roots.

Square root(s) of -16________ Note: every negative number has ___square roots.

Radical Notation: Radical notation does not mean exactly the same thing as square root. It means just the *nonnegative* square root. If $b=\sqrt{a}$ then b is the nonegative number such that $b^2=a$. The symbol is called the radical sign and the term/terms under it are called the radicand.

Examples:

$\sqrt{25}=5 \text{ } only, \text{ } not \pm 5$

$\sqrt{144}=$ ___________

$\sqrt{-81}=$ ___________

Generalizing:

$\sqrt{+}=$ _________________

$\sqrt{-}=$ _________________

Examples that are not perfect squares (Irrational):

$\sqrt{5}$ cannot be simplified, but it can be approximated using a calculator.

$\sqrt{5}$ should be used if an exact answer is required,

$\sqrt{5}\approx 2.2360679775\ldots\approx 2.24$ is an approximation rounded to 2 decimals in this case. No mater how many decimals you write, it is still an approximation because $\sqrt{5}$ is an irrational number.

Simplifying radicals with perfect square variables

Perfect square variable expressions are easily recognized as having an even exponent. When we have a perfect square, we find the square root by dividing the exponent by 2.

$\sqrt{x^4}=x^2$ since $\left(x^2\right)^2=x^4$

$\sqrt{x^{12}}=x^6$

But there's a catch you may not have studied before:

$\sqrt{x^2}\neq x$

$\sqrt{x^6}\neq x^3$

For a radical square root, the answer must ensured to be positive, so $\sqrt{x^2}=|x|$

Examples: $\sqrt{25x^2y^4}$ $\sqrt{100a^4b^6}$

Simplifying Radicals that are not perfect squares but contain perfect square FACTORs:

Property: if a, b ≥ 0 then $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (6.3)

So we break the radicand into perfect square factors and those that are not perfect squares. Then square root the perfect square factors as described above.

Examples:

$\sqrt{8}$ =_______________ $\sqrt{x^7}$ =_______________

$\sqrt{48}$ =_______________ $\sqrt{18a^3b^{10}c^{16}}$ =_______________

$\sqrt{486}$ =_______________

BEYOND SQUARE ROOTS

(part of 6.1)

The ideas you learned about squaring and square roots extend to high powers and roots:

CUBE ROOTS:

Perfect cubes: 1, 8, 27, 64, 125, …

Cube Root:

b is a cube root of a if b^3=a

Examples:

Cube root(s) of 27: __________ Note: every positive number has ____ cube root.

Cube root(s) of -8 _________ Note: every negative number has ____cube root.

Radical Notation: Radical notation in the case of a cube root means the same as the words *cube root* since there is always only one answer. For cube root we put a 3, called the index, to distinguish the notation from square root. If b=$\sqrt[3]{a}$ then b is the number such that b^3=a. The symbol is called the radical sign and the term/terms under it are called the radicand.

Examples:

$\sqrt[3]{125}$ =_____________

$\sqrt[3]{-64}$ = _____________

Generalizing:

$\sqrt[3]{+}$ = ____________________

$\sqrt[3]{-}$ =____________________

Examples that are not perfect cubes (Irrational):

$\sqrt[3]{5}$ cannot be simplified, but it can be approximated using a calculator.

$\sqrt[3]{5}$ should be used if an exact answer is required,

$\sqrt[3]{5} \approx 1.71$ is an approximation rounded to 2 decimals in this case. No mater how many decimals you write, it is still an approximation because it is an irrational number.

Simplifying radicals with perfect cube variables

Perfect cube variable expressions are easily recognized as having an exponent which is a multiple of 3. When we have a perfect cube, we find the cube root by dividing the exponent by 3.

$\sqrt[3]{x^{12}} = x^4$ since $\left(x^4\right)^3 = x^{12}$

$\sqrt[3]{x^9} = x^3$

Good news is there's NO catch with absolute values. The output does NOT have to be forced to be positive so

$\sqrt[3]{x^3} = x$

with NO absolute values

Simplifying Radicals that are not perfect cubes but contain perfect cube FACTORs:

Property: $\sqrt[3]{ab} = \sqrt[3]{a}\,\sqrt[3]{b}$ (6.3)

So we break the radicand into perfect cube factors and those that are not perfect cubes. Then cube root the perfect cube factors as described above.

Examples:

$\sqrt[3]{16} =$ ______________

$\sqrt[3]{-8x^{12}y^{16}} =$ ______________

$\sqrt[3]{-250} =$ ______________

$\sqrt[3]{54a^{10}b^9} =$ ______________

$\sqrt[3]{x^7} =$ ______________

$\sqrt[3]{2000x^{18}y^2} =$ ______________

$\sqrt[3]{x^8} =$ ______________

$\sqrt{27a^3b^5} =$ ______________

(paying attention?)

IN GENERAL, Nth roots

b is an nth root of a if $b^n=a$ and we write $b=\sqrt[n]{a}$

If n is EVEN

"a" has to be nonnegative in order to have a real nth root. If "a" is non-negative it would have two real "nth roots" but the radical notation only yields the positive one. So, if n is EVEN, then like square roots: $\sqrt[n]{+} = +$, $\sqrt[n]{-}$ is undefined and the tricky one… Since for even roots the answer must be positive: $\sqrt[n]{x^n} = |x|$.

If n is ODD, as with cube roots, $\sqrt[n]{+} = +$, $\sqrt[n]{-} = -$, $\sqrt[n]{x^n} = x$, with no absolute values

Simplify using the property: $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$ (if n is even, a, b ≥0 required)

Examples:

$\sqrt[4]{1000x^4y^{12}} =$ ________________ $\sqrt[5]{32x^{10}y^{12}} =$ ________________

$\sqrt[4]{48x^9y^{19}} =$ ________________ $\sqrt[5]{-x^{22}y^3} =$ ________________

$\sqrt[6]{-64x^6y^{12}} =$ ________________ $\sqrt[6]{x^3} =$ ________________(discuss later)

Radical Arithmetic (6.4)

Addition/Subtraction: "Combine Like Terms" : Variable/Radical part must be identical. Combine coefficients without changing variable/radical part.

Examples:

$3\sqrt{5x} + 2\sqrt{5x} =$ ______________ $3x\sqrt{20x} + 2\sqrt{45x^3} =$ __________

$3a\sqrt{ab} - 7a\sqrt{a} - 9a\sqrt{ab} =$ ______________

$7\sqrt[3]{3xy} + 12\sqrt[3]{3xy} - 5\sqrt[3]{3xy} =$ __________ $2\sqrt[4]{a^5b^3} - 4a\sqrt[4]{81a\ b^3} =$ ________

Multiplication: Use earlier property ($\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$) in other direction

Examples:

$\sqrt{2x}\sqrt{6x^5}$ = ______________ $\sqrt[3]{5x^2y^4}\ \sqrt[3]{10x^3y^7}$ = ______________

$(3\sqrt{6a})(5\sqrt{3a})$ = ______________ $\sqrt[4]{8x^3y^5}\ \ \sqrt[4]{10x^3y^2}$ = ______________

$\sqrt{5y}\sqrt{5y}$ = ______________ $\sqrt{2}\ \sqrt[3]{5}$ = ________________(discuss later)

$3\sqrt{x}(2-7\sqrt{x})$ = ______________

$2\sqrt[3]{x^2}(4\sqrt[3]{x}-5)$ = ______________

$(2\sqrt{3}+\sqrt{5})(4\sqrt{2}+1)$ = ____________________

$(\sqrt{a}+3)^2$ = __________________________

$(\sqrt{2x}-\sqrt{5})(\sqrt{2x}+\sqrt{5})$ = __________________________

Division: Use property $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

Examples:

$\dfrac{\sqrt{12}}{\sqrt{3}}$ = ______________

$\dfrac{\sqrt[3]{2x^5}}{\sqrt[3]{54x}}$ = ________________

property in reverse to simplify:

$\sqrt{\dfrac{144}{25}}$ = ______________

$\sqrt[3]{\dfrac{1000x^{10}}{y^6}}$ =

____

More Simplification: Rationalizing the denominator (6.5)

Having an irrational number like $\sqrt{2}$ in the denominator is not considered simplified form. Why?

Examples: (different ways)

$$\frac{1}{\sqrt{2}} \qquad \frac{4}{\sqrt[3]{3}}$$

$$\frac{1}{\sqrt{8}} \qquad \frac{2}{\sqrt[3]{x}}$$

$$\frac{\sqrt{2}}{3\sqrt{5}} \qquad \frac{\sqrt[3]{3}}{\sqrt[3]{x^2y^4}}$$

With two terms in denominator:

$$\frac{3}{2+\sqrt{5}} \qquad \frac{\sqrt{x}+2}{\sqrt{x}-5}$$

Simplifying a few more: Suggestion, if possible combine into one radical and simplify as much as possible first instead of simplifying many radicals and then combining. Usually at this point in the problems you'll notice the instructions say "assume all variable represent positive numbers". That just allows you to disregard absolute values.

$$\sqrt{12x^2y^7z}\ \sqrt{18x^3y^2z} \qquad \frac{\sqrt{8m^3n^7}}{\sqrt{50mn^8}}$$

$$\sqrt[3]{15ab^7}\ \sqrt[3]{25a^{10}b^2} \qquad \frac{\sqrt[3]{5a^5c}}{\sqrt[3]{10ac^2}}$$

RATIONAL EXPONENTS (6.1, 6.2)

$a^{\frac{1}{n}} = \sqrt[n]{a}$ provided the nth root exists (i.e., if n is even, must have a≥0)

Examples:

$25^{\frac{1}{2}} = \sqrt{25} = 5$

$(-16)^{\frac{1}{2}} = \sqrt{-16}$ *is undefined*

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$(-32)^{\frac{1}{5}} = \sqrt[5]{-32} = -2$

We can use this notation for approximating roots with our calculator: To approximate $\sqrt[5]{87}$ with a calculator, we raise 87 to the 1/5 power. Depending on your calculator, you might use a "^" button or a "y^x" button. You will need to put parenthesis around 1/5. E.g.: 87^(1/5) ≈2.88

Simplify: $\left(28x^5y^7z^4\right)^{\frac{1}{2}}$ $\left(-54x^6y^{16}z^{23}\right)^{\frac{1}{3}}$

$$a^{\frac{m}{n}} = \begin{cases} \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m} & m^{th}\ power, then\ n^{th}\ root. \\ \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m & n^{th}\ root, then\ m^{th}\ power.*** \end{cases}$$

provided the nth root exists (i.e., if n is even, must have a≥0)

Example:

$$27^{\frac{2}{3}} = \begin{cases} \left(27^2\right)^{\frac{1}{3}} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9 \\ \left(27^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9 \end{cases}$$

Simplify:

$36^{\frac{3}{2}}$ = ________ $(-144)^{\frac{5}{2}}$ = ________ $\left(-64\right)^{\frac{2}{3}}$ = __________ $32^{-\frac{3}{5}}$ = ________

Simplifying with expressions with rational exponents.

Rules of exponents learned previously apply. Though expressions with rational exponents can always be written in radical form, it is usually easier to work in exponent form.

$\left(5a^{\frac{4}{3}}b^{\frac{1}{2}}\right)\left(7a^{-2}b^{\frac{1}{4}}\right)$ = ___________ $\left(25m^{-2}n^{\frac{1}{4}}\right)^{\frac{3}{2}}$ = __________ $\left(\frac{128a^{4}b^{-3}}{2a^{6}b^{-9}}\right)^{\frac{2}{3}}$ = _____________

easier to simplify inside first

Exponent notation can allow us to simplify some radicals we previously were unable to simplfy. 6.2, 6.3

$\sqrt[6]{x^{3}}$ = ___________________ $\sqrt[9]{y^{3}}$ = _______________

$\sqrt{\sqrt[3]{x^{2}}}$ = ___________________ $\sqrt[3]{x}\,\sqrt[4]{x}$ = ___________________

$\frac{\sqrt[3]{y^{2}}}{\sqrt{y}}$ = ____________________ $\sqrt{2}\,\sqrt[3]{5}$ = _______________