

## Gaussian Elimination

Solve:

$$\begin{cases} x + 2y - 2z = -2 & (1) \\ -5x - 9y + 4z = 3 & (2) \\ 3x + 4y - 5z = -3 & (3) \end{cases} \quad \text{Eliminate } x \quad \left\{ \begin{array}{l} 5Eq(1) \rightarrow 5x + 10y - 10z = -10 \\ Eq(2) \rightarrow -5x - 9y + 4z = 3 \end{array} \right\} \rightarrow ADD \rightarrow y - 6z = -7 \quad (4)$$

$$\left\{ \begin{array}{l} -3Eq(1) \rightarrow -3x - 6y + 6z = 6 \\ Eq(3) \rightarrow 3x + 4y - 5z = -3 \end{array} \right\} \rightarrow ADD \rightarrow -2y + z = 3 \quad (5)$$

$$\text{Elim. } Y: \left\{ \begin{array}{l} 2Eq(4) \rightarrow 2y - 12z = -14 \\ Eq(5) \rightarrow -2y + z = 3 \end{array} \right\} \rightarrow ADD \rightarrow -11z = -11 \quad \text{so } z = 1$$
  

Augmented matrix  $\begin{bmatrix} 1 & 2 & -2 & -2 \\ -5 & -9 & 4 & 3 \\ 3 & 4 & -5 & -3 \end{bmatrix}$  Row operations  $\left\{ \begin{array}{l} 5R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{array} \right\} \Rightarrow \begin{bmatrix} 1 & 2 & -2 & -2 \\ 0 & 1 & -6 & -7 \\ 0 & -2 & 1 & 3 \end{bmatrix}$   $\left\{ \begin{array}{l} 2R_2 + R_3 \rightarrow R_3 \end{array} \right\} \Rightarrow \begin{bmatrix} 1 & 2 & -2 & -2 \\ 0 & 1 & -6 & -7 \\ 0 & 0 & -11 & -11 \end{bmatrix}$   $\left\{ \begin{array}{l} -\frac{1}{11}R_3 \rightarrow R_3 \end{array} \right\} \Rightarrow \boxed{\begin{bmatrix} 1 & 2 & -2 & -2 \\ 0 & 1 & -6 & -7 \\ 0 & 0 & 1 & 1 \end{bmatrix}}$  Row Echelon Form

Now write the corresponding system and use back substitution to solve.:

Example. (This is example 2 pg 664 done differently)

$$\begin{cases} 3x - y + 5z = 14 \\ x + 2y - 2z = 10 \\ x - y + 3z = 4 \end{cases} \quad \begin{bmatrix} 3 & -1 & 5 & 14 \\ 1 & 2 & -2 & 10 \\ 1 & -1 & 3 & 4 \end{bmatrix} \quad R_1 \leftrightarrow R_3 \quad \begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix}$$

Get a "1" in the upper left      Use that "1" to get "0" below