

MATH 5B – SAMPLE FINAL EXAM

SHOW ALL WORK NEATLY AND CLEARLY BOX ALL ANSWERS.

FILL IN THE BLANK WITH THE MOST APPROPRIATE ANSWER. NO PARTIAL CREDIT.

(1) TRUE OR FALSE: If $\lim_{k \rightarrow \infty} a_k = 0$ then $\sum_{k=1}^{\infty} a_k$ converges. _____

(2) $\int \cos^2 x \, dx =$ _____

(3) $\cosh(\ln 3) =$ _____(exactly)

(4) Express the point $(-\sqrt{3}, 1)$ in polar coordinates(exactly)_____

(5) Express the polar point $(8, 7\pi/6)$ in rectangular coordinates (exactly)_____

(6) The derivative of $f(x) = e^{1-3x}$ is _____

(7) $6 - 2 + 2/3 - 2/9 + \dots =$ _____

(8) $\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} =$ _____.

(9) $\int \frac{1}{1-x} \, dx =$ _____

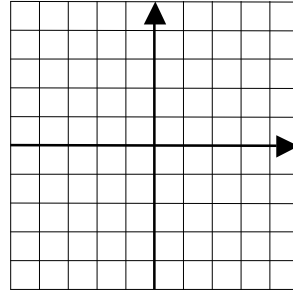
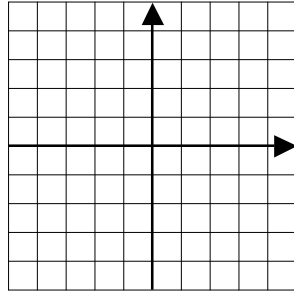
(10) True or False: $1 - \frac{1}{\sqrt[4]{2}} + \frac{1}{\sqrt[4]{3}} - \frac{1}{\sqrt[4]{4}} + \dots$ is a conditionally convergent series._____

(11) For each of the following series, classify as convergent (absolute or conditional if applicable) or divergent. SHOW ALL DETAILS.

(a) $\frac{1}{e} + \frac{2}{e^4} + \frac{3}{e^9} + \frac{4}{e^{16}} + \dots$

(b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n}{4n^2 - 3}$

(12) Sketch the graphs of the polar curves: $r = 3\sin\theta$. and $r = 1+\sin\theta$.



(13) Find each of the following limits. Show details or no credit will be given:

(a) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$

(b) $\lim_{x \rightarrow \infty} (x \tan(1/x))$

(14) If $f(x) = 2 + e^{-x}$, find $f^{-1}(x)$.

(15) Determine the interval of convergence: SHOW DETAILS.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n (n+1)}$$

(16) Compute each of the following integrals. For any given improper integrals, you must first write the integral in terms of the limit as it is defined.

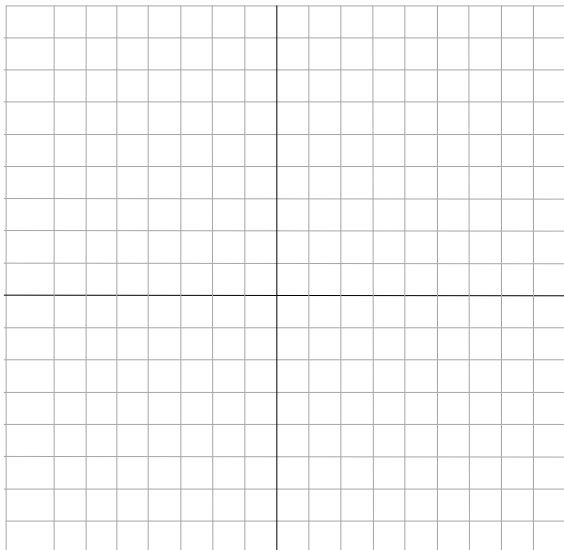
(a) $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$ (b)

(8) $\int \frac{dx}{x^2 \sqrt{x^2 - 25}}$

(c) $\int_0^{\infty} \frac{e^{-x}}{1 + e^{-2x}} dx$

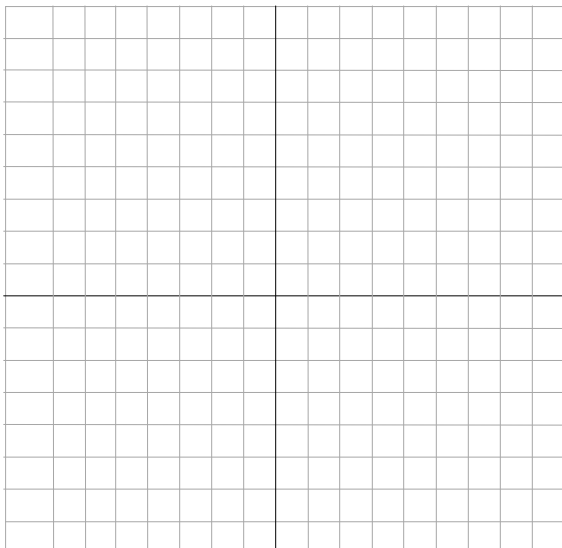
(d) $\int \cos^3 x dx$

- (17) (a) Sketch the curve given by the parametric equations $\begin{cases} x = t^3 \\ y = t^2 \end{cases}$
 (b) Find the length of the portion of the above curve corresponding to $0 \leq t \leq 2$



- (18) Given $f(x) = xe^x$, answer the following, find any other information necessary to obtain a graph and sketch the graph.

- (a) $\lim_{x \rightarrow \infty} f(x) = \text{_____}$
 (b) $\lim_{x \rightarrow -\infty} f(x) = \text{_____}$
 (c) local extrema _____
 (d) Discuss concavity. _____



- (19) (a) Use series to approximate $\int_0^{1/2} \frac{1}{\sqrt{1+x^2}} dx$ correct to 2 decimal places.
- (b) Find the value of the integral exactly by integrating directly.