5A Sample Final Solutions

(1) a) -1 b) -1 c) 0

(2) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3-(x+h)-(x^3-x)}{h} = \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3-h}{h} = \lim_{h\to 0}(3x^2+3xh+h^2-1) = 3x^2-1$

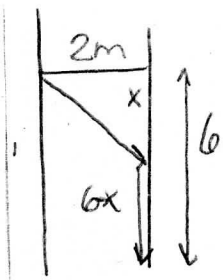
(3) a) $\frac{5x^2+2}{2\sqrt{x}}$ b) $h(x) = (1+\tan^2 x)^3 = (\sec^2 x)^3 = \sec^6 x$
$h'(x) = 6\sec^5 x \sec x \tan x = 6\sec^6 x \tan x$ c) $\frac{1}{(x^2+1)^{3/2}}$

(4) Tangent line slope $y' = x\cos x + \sin x$ At $(\pi, 0)$ $y' = -\pi = m$ Eqn. of line $y-0 = -\pi(x-\pi)$
$y = -\pi x + \pi^2$
(y int π^2)

(5) a) $\frac{3}{16}$ b) $\frac{26}{3}$ c) $\frac{1}{3}(x^2-1)^{3/2} + \sqrt{x^2-1} + C$

(6) $f' = \frac{5-7x}{5(1-x)^{3/5}}$ a) i) $(-\infty, 5/7] \cup [1, \infty)$ ii) $[5/7, 1]$ iii) $(\frac{10}{7}, \infty)$ iv) $(-\infty, 1) \cup (1, \frac{10}{7})$

$f'' = \frac{14x-20}{25(1-x)^{8/5}}$ b) Crit numbers $5/7, 1$ c) inflection $(\frac{10}{7}, f(\frac{10}{7}))$ ≈ 1.02 d) local max $(\frac{5}{7}, \sim 4.32)$
local min $(1, 0)$

(7) 

Minimize Time = Time row + Time walk $= \frac{\sqrt{x^2+4}}{3} + \frac{6-x}{5}$ $0 \le x \le 6$

$T'(x) = \frac{x}{3\sqrt{x^2+4}} - \frac{1}{5} \overset{\text{set}}{=} 0 \Rightarrow x = \frac{3}{2}$

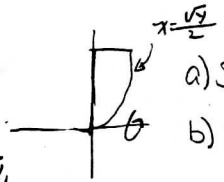
x	$3/2$	0	6
$T(x)$	$\frac{52}{30}$	$\frac{56}{30}$	$\frac{2\sqrt{10}}{3}$

↑ Abs min $\frac{52}{30}$ hr

(8) $f(x) = x - 2\cos x$ $[-\pi, \pi]$
$f'(x) = 1 + 2\sin x$
CRIT $0 = 1 + 2\sin x$
$\sin x = -1/2$ $x = -\frac{5\pi}{6}, -\frac{\pi}{6}$

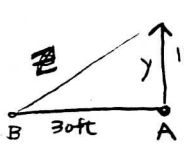
x	$f(x)$	
$-\pi$	$-\pi+2 \approx -1.14$	
π	$\pi+2 \approx 5.14$	MAX
$-\frac{5\pi}{6}$	$-\frac{5\pi}{6}+\sqrt{3} \approx -.89$	
$-\frac{\pi}{6}$	$-\frac{\pi}{6}-\sqrt{3} \approx -2.26$	MIN

(9) 
$A = 2\int_0^2 x^3\,dx = 2\left.\frac{x^4}{4}\right]_0^2 = 8$

(10) 

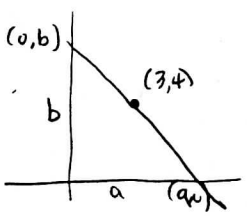
a) Shells $\int_0^{16} 2\pi y \frac{\sqrt{y}}{2}\,dy$
b) Washers $\pi\int_0^2 [16^2 - (4x^2)^2]\,dx$

$y = 4x^2$
$x = \pm\sqrt{y}/4$

(11) 

Given: $\frac{dy}{dt} = 15$
Find $\left.\frac{dz}{dt}\right|_{y=40}$

$z^2 = y^2 + 30^2$
$2z\frac{dz}{dt} = 2y\frac{dy}{dt}$
$\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt}$ so $\left.\frac{dz}{dt}\right|_{y=40} = \frac{40}{50}\cdot 15 = 12$ ft/sec.

(12) 

Minimize Area $= \frac{1}{2}ab$ ---- need to relate a, b.
Slope $= -\frac{b}{a} = \frac{4}{3-a}$ $b = \frac{4a}{a+3}$

$A = \frac{1}{2}a\frac{4a}{a-3} = \frac{2a^2}{a-3}$

Find crit #s $A' = \frac{2a(a-6)}{(a-3)^2}$ $a = 0, 6$ crit ... so $a = 6$ $b = \frac{24}{3} = 8$

Equation $y = -\frac{4}{3}x + 8$

(13) $f(x)$ conts for $x \ge -1/2$ and diffable for $x > -1/2$
So MVT applies in $[0, 4]$
There exists c such that
$f'(c) = \frac{f(4)-f(0)}{4-0}$ $c = 3/2$